SIMPLE ELECTROHYDRODYNAMIC WAVES

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The conditions for the existence and tilting of simple electrohydrodynamic waves are investigated.

In this article we discuss the system of one-dimensional time-dependent electrohydrodynamic (EHD) equations for a polytropic or, in a special case, adiabatic process. We obtain an exact solution for the system of equations in the form of simple (Riemann) waves on the assumption that all the unknowns, in this case the velocity of the medium and the electric field, are functions of the density. The electric field and velocity of the medium are expressed in quadratures in this situation. If the polytropic exponent is an integer, the solution can be written in elementary functions.

Proceeding from the solution thus obtained, we investigate the tilting of a simple wave in electrohydrodynamics. It turns out, in contrast with hydrodynamic and magnetohydrodynamic (MHD) Riemann waves [1-3], that the stated property depends on the combination of the sign of the electric field and the mobility coefficient, as well as on the gasdynamic parameters of the flow.

Strictly speaking, the process of tilting of a Riemann wave can be reduced to the situation where a decrease in the characteristic space scale of the flow necessitates the analysis of charge-diffusion processes and effects associated with hydrodynamic viscosity. In the given situation such an analysis is invalid at the outset, and it is necessary to treat EHD flows with regard for the diffusion of charges when the variation of the electric field is described by the Burgers equation [4, 5].

The exact solutions of the one-dimensional time-dependent system of EHD equations have been studied previously, e.g., in [6] for the case of an incompressible fluid. Processes involving the propagation of weak shocks (first-order discontinuities) in electrohydrodynamics have been investigated in [7], in which their velocities of propagation, representing the velocities of propagation of the leading edge of Riemann waves, were determined.

Riemann-Wave Solution of the EHD Equations

We consider the system of EHD equations, which in the one-dimensional time-dependent case of a polytropic gas flow is reducible to the form [4-9]

$$\begin{gathered}\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+\frac{1}{\rho}\,\frac{\partial p}{\partial x}-\frac{1}{4\pi\rho}E\,\frac{\partial E}{\partial x}=0,\\ \frac{\partial \rho}{\partial t}+\rho\,\frac{\partial u}{\partial x}+u\,\frac{\partial \rho}{\partial x}=0,\ p=A\rho^{n},\\ \frac{\partial E}{\partial t}+(u+bE)\,\frac{\partial E}{\partial x}=0.\end{gathered}\tag{1}$$

We seek solutions of the system (1) in the form of simple waves. Assuming that all the variables depend on a function φ(x, t) and substituting the expressions

$$\frac{\partial f}{\partial t}=\frac{df}{d\varphi}\,\frac{\partial \varphi}{\partial t},\quad \frac{\partial f}{\partial x}=\frac{df}{d\varphi}\,\frac{\partial \varphi}{\partial x},$$

where f is an unknown function, into the system of equations (1), we obtain a homogeneous system of ordinary differential equations. Applying the condition for such a system to have a nontrivial solution, we calculate the possible velocities relative to the fluid particles.

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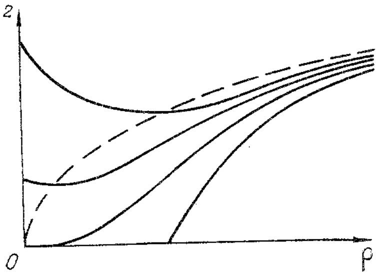

Fig. 1. Qualitative behavior of the integral curves of Eq. (5) for various values of the constant of integration C (initial conditions); the dashed curve represents the null isoclinic line.

By definition, the velocity of a point with constant phase is $a = -\varphi_t'/\varphi_x' - u$. The velocities of propagation are equal to $a_{1,2} = \pm a_0$, $a_0 = (dp/d\rho)^{1/2}$; $a_3 = bE$. These velocities have been calculated previously [4, 5, 7] as the weak-shock propagation velocities.

If now $\varphi(x, t) = \rho(x, t)$, i.e., if all the variables are functions of the density, we obtain the system of equations

$$\frac{du}{d\rho}\frac{\partial\rho}{\partial t} + \left(u\frac{du}{d\rho} + \frac{1}{\rho}\frac{dp}{d\rho} - \frac{1}{4\pi\rho}E\frac{\partial E}{\partial x}\right)\frac{\partial\rho}{\partial x} = 0,$$

$$\frac{\partial\rho}{\partial t} + \left(\rho\frac{du}{d\rho} + u\right)\frac{\partial\rho}{\partial x} = 0, \frac{\partial\rho}{\partial t} + (u + bE)\frac{b\rho}{\partial x} = 0. \tag{2}$$

For the system (2) to have a nontrivial solution it is necessary that the following relations hold:

$$bE\frac{du}{d\rho} + \frac{1}{4\pi\rho}E\frac{dE}{d\rho} = \frac{1}{\rho}\frac{dp}{d\rho}, \tag{3}$$

$$bE = \rho\frac{du}{d\rho}. \tag{4}$$

Multiplying Eq. (3) by ρ and making use of (4), we eliminate the derivative of the velocity u and obtain

$$\frac{d}{d\rho}E^2 + 8\pi b^2E^2 = 8\pi\frac{dp}{d\rho} \equiv 8\pi An\rho^{n-1}. \tag{5}$$

The solution of Eq. (5) can be written in the form

$$E^2 = \exp(-y)\left(B\int y^{n-1}\exp y\,dy + C\right), \tag{6}$$

where $y = 8\pi b^2\rho$; $B = (8\pi)^{1-n}An/b^{2n}$.

We note that with regard for the boundary conditions the solution (6) can be written with a definite integral and expressed in terms of special functions. Then the particle velocity u of the medium is determined in terms of the electric field E by the quadrature

$$u = b\int\frac{E(\rho)}{\rho}d\rho + \text{const.}$$

Asymptotic Representations; Phase Portrait; Special Cases

We use expression (6) and determine the asymptotic relationship of the electric field squared E^2 to the density ρ for $8\pi b^2\rho \ll 1$. Expanding the functions involved in (6) into a series and retaining the first terms of the expansion for $n > 1$, we obtain

$$E^2 = C(1 - 8\pi b^2\rho). \tag{7}$$

To determine the asymptotic behavior for $8\pi b^2\rho \gg 1$ we integrate expression (6) by parts:

$$E^2 = \frac{An}{b^2}\rho^{n-1} - \frac{An(n-1)}{8\pi b^4}\rho^{n-2} + C\exp(-8\pi b^2\rho) + \frac{An(n-1)(n-2)}{64\pi^2b^6}\int\rho^{n-3}\exp(8\pi b^2\rho)\,d\rho. \tag{8}$$

As above, we retain only the principal term of the expansion for $8\pi b^2\rho \gg 1$. According to (8), we have

$$E^2 = \frac{An}{b^2} \rho^{n-1}. \tag{9}$$

This expression is written for the case of greatest interest, where the polytropy exponent $1 < n < 2$. In real EHD flows, where the electric charge carriers are ions, as a rule, the condition $8\pi b^2 \rho \ll 1$ holds. When the charge carriers are electrons, the condition $8\pi b^2 \rho < 1$ or $8\pi b^2 \rho > 1$ can exist (depending on the density of the medium).

The qualitative pattern of behavior of the integral curves of Eq. (5) has the form shown in Fig. 1.

We note that the equation for the null isoclinic line (i.e., the line connecting points at which the slope of the integral path is equal to zero) has the form $E^2 = An\rho^{n-1}/b^2$ and coincides with the asymptotic expression (9) for $8\pi b^2 \rho \gg 1$. This relation can be rewritten in the form $b^2 E^2 = dp/d\rho \equiv a_0^2$.

It can, therefore, be concluded that on the zero isoclinic line the velocity of the leading edge of a Riemann wave propagating in a medium with an electric space charge is equal to the velocity of propagation of small disturbances in the absence of an electric field. The phase plane admits a natural partition into two domains. The integral paths situated above the null isoclinic line correspond to the motions of disturbances with supersonic velocities, and those below the null isoclinic line correspond to subsonic velocities. Paths intersecting the axis $E^2 = 0$ remain at all times in the subsonic domain for $\rho > 0$. Paths intersecting the axis $\rho = 0$ enter the supersonic and the subsonic domain for $E^2 > 0$. This means that for identical boundary conditions corresponding to paths of the latter type, motions with supersonic and subsonic velocities are possible. A wave can be made to go from subsonic to supersonic velocity by decreasing the density of the medium, whereas increasing the density produces the opposite transition.

This separation of the integral paths implies that the boundary conditions are divided into two types: For some, both supersonic and subsonic Riemann waves are possible, while for others only disturbances propagating with subsonic velocities are realized.

In the case of an integer-valued polytropic exponent n the solution (6) is expressed in terms of elementary functions. We make use of expression (8), setting $n = 1$, and obtain $E^2 = An/b^2 + C\exp(-8\pi b^2 \rho)$. Similarly, for $n = 2$ we have

$$E^2 = C\exp(-8\pi b^2 \rho) + \frac{An\rho}{b^2} - \frac{An(n-1)}{8\pi b^4}. \tag{10}$$

Conditions for the Tilting of a Riemann Wave

To analyze the wave deformation with time we calculate the derivative of the wave phase velocity $\lambda(\rho)$ with respect to the density, where $\lambda(\rho) = u(\rho) + bE(\rho)$.

In the general case the tilting condition for a simple wave has the form [1, 2]

$$\frac{d\lambda}{d\rho} > 0. \tag{11}$$

In application to the investigated EHD Riemann wave condition (11) can be rewritten as follows on the basis of Eq. (4) and the fact that $\rho > 0$:

$$bE + b\rho \frac{dE}{d\rho} > 0. \tag{12}$$

Substituting into (12) the expression for $dE/d\rho$ from Eq. (5) and multiplying by the positive quantity E^2/ρ, after suitable transformations we have

$$bE\left(a_0^2 + \frac{E^2}{4\pi\rho} - b^2 E^2\right) > 0. \tag{13}$$

Thus, tilting of the EHD Riemann wave takes place under the conditions

$$bE > 0,\ a_0^2 + \frac{E^2}{4\pi\rho} - b^2 E^2 > 0. \tag{14}$$

$$bE < 0,\ a_0^2 + \frac{E^2}{4\pi\rho} - b^2E^2 < 0. \tag{15}$$

We note that the terms in the parentheses in expression (13) represent velocities squared. The first term a_0^2 is the square of the velocity of small hydrodynamic disturbances, b^2E^2 is the square of the velocity of disturbances propagating via charged particles, and the middle term $E^2/4\pi\rho$ is the square of a velocity analogous to the Alfvén velocity in magnetohydrodynamics. Inequality (14) shows that for the tilting of subsonic EHD Riemann waves the following condition is sufficient:

$$bE > 0. \tag{16}$$

Two cases are possible for supersonic waves. If the wave is supersonic, but $a_0^2 + E^2/4\pi\rho > b^2E^2$, i.e., the wave is comparatively slow, then the tilting condition coincides with (16). When the velocity of a wave propagating via charges is sufficiently large, i.e., $b^2E^2 > a_0^2 + E^2/4\pi\rho$, the tilting process occurs under the condition $bE < 0$.

In real EHD flows, where the medium is acted upon by a subbreakdown electric field (the EHD equations are also applicable in this case), for charged media with an ionic charge component, as a rule, the condition $b^2E^2 < a_0^2 + E^2/4\pi\rho$ always holds. On the other hand, for flows in which the charge carriers are electrons, the true condition is $b^2E^2 > a_0^2 + E^2/4\pi\rho$.

In contrast with hydrodynamic and MHD Riemann waves, EHD Riemann waves have a distinctive feature in that a steady-state (time-invariant) Riemann wave can exist. Its existence requires satisfaction of the condition

$$a_0^2 + \frac{E^2}{4\pi\rho} - b^2E^2 = 0.$$

The use of asymptotic expressions in the limits of large and small densities ρ simplifies the tilting condition (13). For relatively large densities, i.e., for $8\pi b^2\rho \gg 1$, by substituting expression (8) into (13) we arrive at condition (16). We infer from a joint analysis of (7) and (13) that the tilting of a Riemann wave for $8\pi b^2\rho \ll 1$ is associated with the sign of the constant of integration in the solution (6), i.e., actually depends on the initial conditions. Tilting takes place for $bEC > 0$.

NOTATION

A, constant in the polytropic equation; a_0, propagation velocity of small disturbances in zero electric field; $a_{1,2,3}$, characteristic velocities; B, grouped constants; b, mobility coefficient of charged particles; C, constant of integration; E, electric field strength; f, unknown function; n, polytropic exponent; x, coordinate; y, dimensionless density; u, p, ρ, velocity, pressure, and density of the medium; t, time; λ, wave phase velocity; $\pi = 3.14...$; φ, independent variable for a simple wave.

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TRANSPORT PROPERTIES OF NITROGEN, OXYGEN, CARBON DIOXIDE, AND AIR AT LOW DENSITIES AND TEMPERATURES FROM 50 TO 3000°K

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We calculate the viscosity and thermal conductivity, the Prandtl number, and the Eucken factor for a (12-7, δ) pair model potential. The calculated values agree with correlated experimental data within the limits of error of the measurements.

The Chapman—Enskog theory establishes a functional relation between the transport coefficients of a rarefied monatomic gas and the pair potential energy (potential) of the interparticle interaction [1, 2]. The calculation of the potential energy of the interparticle interaction over a wide range of distances is extremely difficult. Therefore, model potentials are generally used [2, 3]. However, the known pair potential models of the interaction (Lennard-Jones, Buckingham, Kihara, etc.) are unsuitable for calculations, since the principle of corresponding states [4] is not actually satisfied for them.

It was shown in [5, 6] that the (12-7) two-parameter pair model potential

$$\varphi(r) = 5.1042\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{7}\right], \tag{1}$$

where ε is the depth of the potential well and σ is the molecular diameter, gives a consistent description of various experimental data on the properties of monatomic gases at low densities; i.e., for it the principle of corresponding states is satisfied. This permitted the calculation of transport coefficients of monatomic gases and binary mixtures of them for low densities and temperatures from 100 to 6000°K [7]. Subsequent measurements [8] confirmed the accuracy of the calculations.

A generalization of the pair potential (1) was proposed in [9, 10] for nonpolar polyatomic molecules

$$\varphi(r) = \begin{cases} \infty & r \leqslant r_e \\ 5.1042\varepsilon\left[\left(\dfrac{\sigma^2 - r_e^2}{r^2 - r_e^2}\right)^{6} - \left(\dfrac{\sigma^2 - r_e^2}{r^2 - r_e^2}\right)^{7/2}\right], & r \geqslant r_e, \end{cases} \tag{2}$$

where r_e is the distance between the outer atoms forming the core of the molecule. It was shown that the three-parameter pair model potential (2) gives a consistent description of various experimental data on the properties of nonpolar polyatomic gases whose molecules have very different geometric structures [9]. The Kong combining rules [11] were generalized in [10] to the case of the potential (2), which permitted the calculation of thermodynamic properties of nonpolar polyatomic gases and mixtures of them for low and medium densities over a wide range of temperatures.

The present article presents calculated values of transport coefficients of air and its components (nitrogen, oxygen, carbon dioxide). The viscosity was calculated with the Chapman—Enskog theory, taking account of the Kihara correction in higher approximations [1, 2]. The thermal conductivity was calculated by using results of the nonlinearized Mason—Monchik theory [12], refined by Antye [13]:

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